COMPOUND ANGLES - 5.4

Q - 1

01. $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$

02. $\cos\frac{\pi}{12} + \cos\frac{3\pi}{12} + \cos\frac{9\pi}{12} + \cos\frac{11\pi}{12} = 0$

03. $\sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\frac{11\pi}{15} - \sin\frac{14\pi}{15} = 0$

Q - 2

01. $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8} = 2$

02. $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{9\pi}{14} + \sin^2\frac{6\pi}{7} = 2$

03. $\cos^2\frac{\pi}{10} + \cos^2\frac{2\pi}{5} + \cos^2\frac{3\pi}{5} + \cos^2\frac{9\pi}{10} = 2$

Q - 3

01. $\cos^2\left(\frac{\pi}{4} - x\right) + \cos^2\left(\frac{\pi}{4} + x\right) = 1$

02. $\sin^2\left(\frac{\pi}{4} - x\right) + \sin^2\left(\frac{\pi}{4} + x\right) = 1$

ALLIED ANGLES

Q - 4

01. $\cos\theta + \sin(270^o + \theta) - \sin(270^o - \theta) + \cos(180^o + \theta) = 0$

02. $\frac{\sin(90^o + A)}{\cos(-A)} - \frac{\sin(180^o - A)}{\sin(-A)} + \frac{\tan(270^o + A)}{\cot(-A)} = 3$

03. $\frac{\operatorname{cosec}(90^o - A)\,.\,\sin(180^o - A)\,.\,\cot(360^o - A)}{\sec(180^o + A)\,.\,\tan(90^o + A)\,.\,\sin(-A)} = 1$

04. $\frac{\cos(90^o + \theta)\,.\,\sec(270^o + \theta)\,.\,\sin(180^o + \theta)}{\cos(-\theta)\,.\,\cos(270^o - \theta)\,.\,\tan(180^o + \theta)} = -\operatorname{cosec}\theta$

Q - 5

01. $\cos 24^o + \cos 55^o + \cos 125^o + \cos 204^o + \cos 300^o = 1/2$

02. $\cos 510^o\,.\,\cos 330^o - \sin 390^o\,.\,\cos 120^o = -1/2$

03. $\sin(-330^o)\,.\cos(-300^o) + \sin(-420^o).\cos 390^o = -1/2$

04. $\cot(405^o)\,.\,\tan(-495^o) - \tan 585^o\,.\,\cot(-495^o) = 0$

SOLUTION SET

Q - 1

01. $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$

$$\text{LHS} = \cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8}$$

$$= \cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\left(\frac{8\pi - 3\pi}{8}\right) + \cos\left(\frac{8\pi - \pi}{8}\right)$$

$$= \cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\left(\pi - \frac{3\pi}{8}\right) + \cos\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos\frac{\pi}{8} + \cos\frac{3\pi}{8} - \cos\frac{3\pi}{8} - \cos\frac{\pi}{8} \qquad \text{USING } \cos(\pi - \theta) = -\cos\theta$$

$$= 0$$

02. $\cos\frac{\pi}{12} + \cos\frac{3\pi}{12} + \cos\frac{9\pi}{12} + \cos\frac{11\pi}{12} = 0$

$$\text{LHS} = \cos\frac{\pi}{12} + \cos\frac{3\pi}{12} + \cos\frac{9\pi}{12} + \cos\frac{11\pi}{12}$$

$$= \cos\frac{\pi}{12} + \cos\frac{3\pi}{12} + \cos\left(\frac{12\pi - 3\pi}{12}\right) + \cos\left(\frac{12\pi - \pi}{12}\right)$$

$$= \cos\frac{\pi}{12} + \cos\frac{3\pi}{12} + \cos\left(\pi - \frac{3\pi}{12}\right) + \cos\left(\pi - \frac{\pi}{12}\right)$$

$$= \cos\frac{\pi}{12} + \cos\frac{3\pi}{12} - \cos\frac{3\pi}{12} - \cos\frac{\pi}{12} \qquad \text{USING } \cos(\pi - \theta) = -\cos\theta$$

$$= 0$$

03. $\sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\frac{11\pi}{15} - \sin\frac{14\pi}{15} = 0$

$$\text{LHS} = \sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\frac{11\pi}{15} - \sin\frac{14\pi}{15}$$

$$= \sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\left(\frac{15\pi - 4\pi}{15}\right) - \sin\left(\frac{15\pi - \pi}{15}\right)$$

$$= \sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\left(\pi - \frac{4\pi}{15}\right) - \sin\left(\pi - \frac{\pi}{15}\right)$$

$$= \sin\frac{\pi}{15} + \sin\frac{4\pi}{15} - \sin\frac{4\pi}{15} - \sin\frac{\pi}{15} \qquad \text{USING } \sin(\pi - \theta) = \sin\theta$$

$$= 0$$

01. $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8} = 2$

Q-2

LHS

$$= \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$$

$$= \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\left(\frac{4\pi + \pi}{8}\right) + \sin^2\left(\frac{4\pi + 3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{2} + \frac{3\pi}{8}\right)$$

$$= \sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}$$ USING $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$$= \left[\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right] + \left[\sin^2\frac{3\pi}{8} + \cos^2\frac{3\pi}{8}\right]$$

$$= 1 + 1$$

$$= 2$$

02. $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{9\pi}{14} + \sin^2\frac{6\pi}{7} = 2$

LHS

$$= \sin^2\frac{2\pi}{14} + \sin^2\frac{5\pi}{14} + \sin^2\frac{9\pi}{14} + \sin^2\frac{12\pi}{14}$$

$$= \sin^2\frac{2\pi}{14} + \sin^2\frac{5\pi}{14} + \sin^2\left(\frac{7\pi + 2\pi}{14}\right) + \sin^2\left(\frac{7\pi + 5\pi}{14}\right)$$

$$= \sin^2\frac{2\pi}{14} + \sin^2\frac{5\pi}{14} + \sin^2\left(\frac{\pi}{2} + \frac{2\pi}{14}\right) + \sin^2\left(\frac{\pi}{2} + \frac{5\pi}{14}\right)$$

$$= \sin^2\frac{2\pi}{14} + \sin^2\frac{5\pi}{14} + \cos^2\frac{2\pi}{14} + \cos^2\frac{5\pi}{14}$$ USING $\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$$= \left[\sin^2\frac{2\pi}{14} + \cos^2\frac{2\pi}{14}\right] + \left[\sin^2\frac{5\pi}{14} + \cos^2\frac{5\pi}{14}\right]$$

$$= 1 + 1$$

$$= 2$$

03. $\cos^2\frac{\pi}{10} + \cos^2\frac{2\pi}{5} + \cos^2\frac{3\pi}{5} + \cos^2\frac{9\pi}{10} = 2$

LHS $= \cos^2\frac{\pi}{10} + \cos^2\frac{4\pi}{10} + \cos^2\frac{6\pi}{10} + \cos^2\frac{9\pi}{10}$

$= \cos^2\frac{\pi}{10} + \cos^2\frac{4\pi}{10} + \cos^2\left(\frac{5\pi + \pi}{10}\right) + \cos^2\left(\frac{5\pi + 4\pi}{10}\right)$

$= \cos^2\frac{\pi}{10} + \cos^2\frac{4\pi}{10} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{10}\right) + \cos^2\left(\frac{\pi}{2} + \frac{4\pi}{10}\right)$

$= \cos^2\frac{\pi}{10} + \cos^2\frac{4\pi}{10} + \sin^2\frac{\pi}{10} + \sin^2\frac{4\pi}{10}$ USING $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

$= \left[\cos^2\frac{\pi}{10} + \sin^2\frac{\pi}{10}\right] + \left[\cos^2\frac{4\pi}{10} + \sin^2\frac{4\pi}{10}\right]$

$= 1 + 1$

$= 2$

Q-3

01. $\cos^2\left(\frac{\pi}{4} - x\right) + \cos^2\left(\frac{\pi}{4} + x\right)$

let

$\frac{\pi}{4} - x = \theta \quad \therefore \quad x = \frac{\pi}{4} - \theta$

$\cos^2\left(\frac{\pi}{4} - x\right) + \cos^2\left(\frac{\pi}{4} + x\right)$

$= \cos^2\theta + \cos^2\left(\frac{\pi}{4} + \frac{\pi}{4} - \theta\right)$

$= \cos^2\theta + \cos^2\left(\frac{\pi}{2} - \theta\right)$

$= \cos^2\theta + \sin^2\theta$

$= 1$

02. $\sin^2\left(\frac{\pi}{4} - x\right) + \sin^2\left(\frac{\pi}{4} + x\right)$

let

$\frac{\pi}{4} - x = \theta \quad \therefore \quad x = \frac{\pi}{4} - \theta$

$\sin^2\left(\frac{\pi}{4} - x\right) + \sin^2\left(\frac{\pi}{4} + x\right)$

$= \sin^2\theta + \sin^2\left(\frac{\pi}{4} + \frac{\pi}{4} - \theta\right)$

$= \sin^2\theta + \sin^2\left(\frac{\pi}{2} - \theta\right)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Q-4

01. $\cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) = 0$

$$\begin{aligned}
\text{LHS} &= \cos\theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) \\
&= \cos\theta + (-\cos\theta) - (-\cos\theta) + (-\cos\theta) \\
&= \cos\theta - \cos\theta + \cos\theta - \cos\theta \\
&= 0
\end{aligned}$$

02. $\dfrac{\sin(90^\circ + A)}{\cos(-A)} - \dfrac{\sin(180^\circ - A)}{\sin(-A)} + \dfrac{\tan(270^\circ + A)}{\cot(-A)} = 3$

LHS

$$\begin{aligned}
&= \frac{\sin(90^\circ + A)}{\cos(-A)} - \frac{\sin(180^\circ - A)}{\sin(-A)} + \frac{\tan(270^\circ + A)}{\cot(-A)} \\
&= \frac{\cos A}{\cos A} - \frac{\sin A}{-\sin A} + \frac{-\cot A}{-\cot A} \\
&= 1 + 1 + 1 \\
&= 3
\end{aligned}$$

03. $\dfrac{\operatorname{cosec}(90^\circ - A) \,.\, \sin(180^\circ - A) \,.\, \cot(360^\circ - A)}{\sec(180^\circ + A) \,.\, \tan(90^\circ + A) \,.\, \sin(-A)} = 1$

LHS

$$\begin{aligned}
&= \frac{\operatorname{cosec}(90^\circ - A) \,.\, \sin(180^\circ - A) \,.\, \cot(360^\circ - A)}{\sec(180^\circ + A) \,.\, \tan(90^\circ + A) \,.\, \sin(-A)} \\
&= \frac{\sec A \,.\, \sin A.\, (-\cot A)}{(-\sec A) \,.\, (-\cot A).\, (-\sin A)} \\
&= 1
\end{aligned}$$

04. $\dfrac{\cos(90^\circ + \theta) \,.\, \sec(270^\circ + \theta) \,.\, \sin(180^\circ + \theta)}{\cos(-\theta) \,.\, \cos(270^\circ - \theta) \,.\, \tan(180^\circ + \theta)} = -\operatorname{cosec}\theta$

$$\begin{aligned}
&= \frac{\cancel{(-\sin\theta)} \,.\, (\operatorname{co}\cancel{\sec\theta) \,.\, (-\sin\theta})}{\cos\theta \,.\, \cancel{(-\sin\theta)} \,.\, \tan\theta} \\
&= -\frac{1}{\cos\theta \,.\tan\theta} \\
&= -\frac{1}{\cancel{\cos\theta} \,.\dfrac{\sin\theta}{\cancel{\cos\theta}}} \\
&= -\operatorname{cosec}\theta.
\end{aligned}$$

Q - 5

01. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = {}^1/_2$

$\cos 125 = \cos(180 - 55) = -\cos 55$

$\cos 204 = \cos(180 + 24) = -\cos 24$

$\cos 300 = \cos(360 - 60) = +\cos 60 = {}^1/_2.$

Now

$\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = {}^1/_2$

$= \cos 24^\circ + \cos 55^\circ - \cos 55^\circ - \cos 24^\circ + {}^1/_2$

$= {}^1/_2$

02. $\cos 510^\circ . \cos 330^\circ - \sin 390^\circ . \cos 120^\circ = -1/2$

$\cos 510 = \cos(540 - 30) = \cos(3\text{x}180 - 30) = -\cos 30 = -{}^{\sqrt{3}}/_2$

$\cos 330 = \cos(360 - 30) = \cos(2\text{x}180 - 30) = +\cos 30 = +{}^{\sqrt{3}}/_2$

$\sin 390 = \sin(360 + 30) = \sin(2\text{x}180 + 30) = +\sin 30 = +{}^1/_2$

$\cos 120 = \cos(180 - 60) = \quad = -\cos 60 = -{}^1/_2$

$\cos 510^\circ . \cos 330^\circ - \sin 390^\circ . \cos 120^\circ$

$= \frac{-\sqrt{3}}{2} \text{ x } \frac{\sqrt{3}}{2} - \frac{1}{2} \text{ x } \frac{-1}{2}$

$= \frac{-3}{4} + \frac{1}{4}$

$= \frac{-1}{2}$

03. $\sin(-330^\circ) . \cos(-300^\circ) + \sin(-420^\circ) . \cos 390^\circ = -1/2$

$\sin(-330) = -\sin(330) = -\sin(360 - 30) = -\sin(2\text{x}180 - 30) = \sin 30 = +{}^1/_2$

$\cos(-300) = \cos 300 = \cos(2\text{x}180 - 60) = +\cos 60 = +{}^1/_2$

$\sin(-420) = -\sin(420) = -\sin(360 + 60) = -\sin(2\text{x}180 + 60) = -\sin 60 = -{}^{\sqrt{3}}/_2$

$\cos 390 = \cos(360 + 30) = \cos(2\text{x}180 + 30) = +\cos 30 = +{}^{\sqrt{3}}/_2$

$\sin(-330^\circ) . \cos(-300^\circ) + \sin(-420^\circ) . \cos 390^\circ$

$= \frac{1}{2} \text{ x } \frac{1}{2} + \frac{-\sqrt{3}}{2} \text{ x } \frac{\sqrt{3}}{2}$

$= \frac{1}{4} - \frac{3}{4}$

$= \frac{-1}{2}$

04. $\cot(405^\circ) \cdot \tan(-495^\circ) - \tan 585^\circ \cdot \cot(-495^\circ) = 0$

$\cot(405) = \cot(360 + 45) = \cot(2\text{x}180 + 45) = +\cot 45 = +1$

$\tan(-495^\circ) = -\tan 495 = -\tan(540 - 45) = -\tan(3\text{x}180 - 45) = -(-\tan 45) = +1$

$\tan(585) = \tan(540 + 45) = \tan(3\text{x}180 + 45) = +\tan 45 = +1$

$\cot(-495^\circ) = -\cot 495 = -\cot(540 - 45) = -\cot(3\text{x}180 - 45) = -(-\cot 45) = +1$

$\cot(405^\circ) \cdot \tan(-495^\circ) - \tan 585^\circ \cdot \cot(-495^\circ)$

$= (1)(1) - (1)(1)$

$= 0$